Pulse Design for UWB Systems

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Introduction

- Motivations
  - Ultra wideband (UWB) radios have attracted increasing for their potential applications in short range high data rate wireless communications.
  - The temporal diversity makes UWB technology a promising alternative for robust wireless indoor communications.

Introduction

- Problem Definitions
  - UWB signals may cause interferences to other systems, such as GPS systems.
  - The Federal Communications Commission (FCC) has restricted UWB signals to be less than -10dB and -20dB within the frequency range between 3.1GHz and 10.6GHz, respectively. Also, the pulse width is required to be less than 1ns.
  - The aim of our work is to design a pulse for UWB systems such that the pulse strictly satisfies the specifications defined in both the time and frequency domains.

Introduction

- Existing Methods
  - Gaussian monocycle pulse
    - Gaussian pulse satisfies the uncertainty principle.

$$\frac{1}{2\pi}\int_{-\infty}^{+\infty} t^2 \left|g(t)\right|^2 dt \int_{-\infty}^{+\infty} \omega^2 \left|G(\omega)\right|^2 d\omega = \frac{1}{4}$$

    - Derivative of Gaussian pulse fails to satisfy both the time and frequency specifications.

Introduction

- Existing Methods
  - Filtering or multiplying a window on the Gaussian monocycle pulse
    - Filtering the pulse would increase the pulse duration and therefore reduces the data rate.
    - Multiplying a window on the pulse would increase the bandwidth of the pulse.
    - This method is converted the pulse design problem to a filter or window design problem. Since the design is based on the processing of the Gaussian monocycle pulse, the designed pulse is not optimal.
    - The designed pulse is not guaranteed to be satisfied both the time and frequency specifications.

Introduction

- Existing Methods
  - Modified Hermite polynomials, sinusoidal signals and Battle-Lemarie wavelets
    - The problem becomes the determination of the linear combinations of the basis signals.
    - Since the pulse is designed based on the processing of these basis signals, the designed pulse is still not optimal.
    - The designed pulse is still not guaranteed to be satisfied both the time and frequency specifications.
    - Increasing the number of coefficients would increase the hardware implementation costs.

Introduction

- Challenges
  - The specifications suggested by FCC are defined in both time and frequency domains, which are continuous sets.
  - Since a continuous set consists of an infinite number of discrete elements, the designed pulse requires to satisfy an infinite number of constraints.
  - Existing algorithms only solve optimization problems with finite number of constraints. How to guarantee the obtained solution would satisfy an infinite number of constraints?

Introduction

- Challenges
  - How to design a pulse without assuming a particular set of basis signals?
  - How to formulate the optimization problems such that the constraints in both time and frequency domain are captured in the design?

Proposed Method

- Problem Formulation
  - Idea: Instead of designing a pulse, we design a rational, causal and proper infinite impulse response (IIR) filter and employ the impulse response of the designed filter as the pulse for UWB systems.
  - Advantage: Without assuming a particular set of basis signals.

Proposed Method

- Problem Formulation
  - Notations
    - Filter order: N
    - Numerator coefficients: b_n for $n = 1,2,\cdots,N$
    - Denominator coefficients: a_n for $n = 1,2,\cdots,N$
    - Vector containing numerator coefficients: $\mathbf{b} \equiv \begin{bmatrix} b_1 & \cdots & b_N \end{bmatrix}^T$
    - Vector containing denominator coefficients: $\mathbf{a} \equiv \begin{bmatrix} a_1 & \cdots & a_N \end{bmatrix}^T$
    - Impulse response of the designed pulse: $\psi(t)$
    - Frequency response of the designed pulse: $\Psi(\omega)$
    - Numerator polynomial of the frequency response: $P(\omega)$
    - Denominator polynomial of the frequency response: $S(\omega)$
    - Impulse response of $\frac{1}{S(\omega)}$: $p(t)$
    - Desired frequency response of the pulse: $D(\omega)$

Proposed Method

- Problem Formulation
  - Define $\boldsymbol{\xi}(\omega) \equiv \left[j\omega \quad \cdots \quad (j\omega)^N \right]^T$
  - Then

$$\Psi(\omega) = \frac{P(\omega)}{S(\omega)} = \frac{\sum_{n=1}^{N} b_n (j\omega)^n}{1 + \sum_{n=1}^{N} a_n (j\omega)^n} = \frac{\boldsymbol{\xi}^T(\omega)\mathbf{b}}{1 + \boldsymbol{\xi}^T(\omega)\mathbf{a}}$$

  - Define the error function in frequency domain as: $E_f(\omega) \equiv \left| \boldsymbol{\xi}^T(\omega)\mathbf{b} - D(\omega)\left(1 + \boldsymbol{\xi}^T(\omega)\mathbf{a}\right) \right|^2$
  - $E_f(\omega)$ reflects the modulus square of the difference between the designed and the desired frequency responses of the pulse.

Proposed Method

- Problem Formulation
  - Define B and T as the interested band and the time support region of the pulse.
  - B is the frequency range from 3.1GHz and 10.6GHz. T is the time support region from 0s to 1ns.
  - Denote the acceptable bounds of $E_f(\omega)$ outside B and that of $|\psi(t)|$ outside T as δ_f and δ_t, respectively.

Proposed Method

- Problem Formulation
  - Our objective is to design a pulse which minimizes the total area of $E_f(\omega)$ subject to $|\psi(t)| \le \delta_t \quad \forall t \in \Re \setminus T$ and $E_f(\omega) \le \delta_f \quad \forall \omega \in \Re \setminus B$.
  - Since an IIR is designed, the stability condition is also required. That is: $\mathrm{Re}\left(1 + \boldsymbol{\xi}^T(\omega)\mathbf{a}\right) < 0 \quad \forall \omega \in \Re$.

Proposed Method

- Problem Formulation
  - Problem $(\mathbf{P}_f)$

$$\min_{(\mathbf{a},\mathbf{b})} J(\mathbf{a},\mathbf{b}) \equiv \int_{-\infty}^{+\infty} E_f(\omega)\, d\omega$$

subject to
$$|\psi(t)| \le \delta_t \quad \forall t \in \Re \setminus T$$
$$E_f(\omega) \le \delta_f \quad \forall \omega \in \Re \setminus B$$
$$\mathrm{Re}\left(1 + \boldsymbol{\xi}^T(\omega)\mathbf{a}\right) < 0 \quad \forall \omega \in \Re$$

Proposed Method

- Problem Formulation
  - Since

$$E_f(\omega) = \mathbf{b}^T \xi^*(\omega)\xi^T(\omega)\mathbf{b} + \mathbf{a}^T D^*(\omega)D(\omega)\xi^*(\omega)\xi^T(\omega)\mathbf{a} + 2D^*(\omega)D(\omega)\mathrm{Re}\left(\xi^T(\omega)\right)\mathbf{a}$$

$$-2\,\mathrm{Re}\left(D^*(\omega)\xi^T(\omega)\right)\mathbf{b} + D^*(\omega)D(\omega) - \mathbf{b}^T D(\omega)\xi^*(\omega)\xi^T(\omega)\mathbf{a} - \mathbf{a}^T D^*(\omega)\xi^*(\omega)\xi^T(\omega)\mathbf{b}$$

  - By denoting

$$\mathbf{x} = \begin{bmatrix}\mathbf{a}^T & \mathbf{b}^T\end{bmatrix}^T$$

$$\mathbf{Q}_{bb} \equiv 2\int_{-\infty}^{+\infty} \xi^*(\omega)\xi^T(\omega)d\omega$$

$$\mathbf{Q}'_{bb}(\omega) \equiv 2\xi^*(\omega)\xi^T(\omega)$$

$$\mathbf{Q}_{aa} \equiv 2\int_{-\infty}^{+\infty} D^*(\omega)D(\omega)\xi^*(\omega)\xi^T(\omega)d\omega$$

$$\mathbf{Q}'_{aa}(\omega) \equiv 2D^*(\omega)D(\omega)\xi^*(\omega)\xi^T(\omega)$$

$$\mathbf{Q}_{ba} \equiv -2\int_{-\infty}^{+\infty} D(\omega)\xi^*(\omega)\xi^T(\omega)d\omega$$

$$\mathbf{Q}'_{ba}(\omega) \equiv -2D(\omega)\xi^*(\omega)\xi^T(\omega)$$

$$\mathbf{Q}_{ab} \equiv -2\int_{-\infty}^{+\infty} D^*(\omega)\xi^*(\omega)\xi^T(\omega)d\omega$$

$$\mathbf{Q}'_{ab}(\omega) \equiv -2D^*(\omega)\xi^*(\omega)\xi^T(\omega)$$

Proposed Method

- Problem Formulation

$$\mathbf{Q} \equiv \begin{bmatrix} \mathbf{Q}_{aa} & \mathbf{Q}_{ab} \\ \mathbf{Q}_{ba} & \mathbf{Q}_{bb} \end{bmatrix}$$

$$\mathbf{Q}'(\omega) \equiv \begin{bmatrix} \mathbf{Q}'_{aa}(\omega) & \mathbf{Q}'_{ab}(\omega) \\ \mathbf{Q}'_{ba}(\omega) & \mathbf{Q}'_{bb}(\omega) \end{bmatrix}$$

$$\mathbf{c}_a \equiv 2\int_{-\infty}^{+\infty} D^*(\omega)D(\omega)\operatorname{Re}(\xi(\omega))d\omega$$

$$\mathbf{c}_b \equiv -2\int_{-\infty}^{+\infty} \operatorname{Re}\left(D^*(\omega)\xi(\omega)\right)d\omega$$

$$\mathbf{c}'_a(\omega) \equiv 2D^*(\omega)D(\omega)\operatorname{Re}(\xi(\omega))$$

$$\mathbf{c}'_b(\omega) \equiv -2\operatorname{Re}\left(D^*(\omega)\xi(\omega)\right)$$

$$\mathbf{c} \equiv \begin{bmatrix} \mathbf{c}_a^T & \mathbf{c}_b^T \end{bmatrix}^T$$

$$\mathbf{c}'(\omega) \equiv \begin{bmatrix} \mathbf{c}_a'^T(\omega) & \mathbf{c}_b'^T(\omega) \end{bmatrix}^T$$

$$p \equiv \int_{-\infty}^{+\infty} D^*(\omega)D(\omega)d\omega$$

$$p'(\omega) \equiv D^*(\omega)D(\omega) - \delta_f$$

Proposed Method

- Problem Formulation
  - Then Problem $(\mathbf{P}_f)$ becomes the following problem:
  - Problem $(\mathbf{P}'_f)$

$$\min_{\mathbf{x}} J(\mathbf{x}) \equiv \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{c}^T\mathbf{x} + p$$

subject to $|\psi(t)| \le \delta_t \quad \forall t \in \Re \setminus T$

$$\frac{1}{2}\mathbf{x}^T\mathbf{Q}'(\omega)\mathbf{x} + \mathbf{c}'^T(\omega)\mathbf{x} + p'(\omega) \le 0 \quad \forall \omega \in \Re \setminus B$$

$$\operatorname{Re}\left(1 + \begin{bmatrix}\boldsymbol{\xi}^T(\omega) & \mathbf{0}\end{bmatrix}\mathbf{x}\right) < 0 \quad \forall \omega \in \Re$$

Proposed Method

- Problem Formulation
  - Denote the poles of $\frac{1}{S(\omega)}$ as r_n for $n = 1,2,\cdots,N$

    That is: $S(\omega) = \prod_{n=1}^{N}\left(1 + \frac{j\omega}{r_n}\right)$

    Then

$$p(t) = \sum_{j=1}^{N} \frac{e^{-r_n t}\prod_{n=1}^{N} r_n}{\prod_{\substack{n=1\\ n\neq j}}^{N}(r_n + j\omega)} \quad \forall t \geq 0$$

    Since $\Psi(\omega) = \frac{\boldsymbol{\xi}^T(\omega)\mathbf{b}}{S(\omega)}$

    we have $\psi(t) = \sum_{n=1}^{N} b_n \frac{d^n p(t)}{dt^n}$

Proposed Method

- Problem Formulation
  - Define $\boldsymbol{\rho}(\mathbf{x},t) \equiv \left[\dfrac{dp(t)}{dt} \quad \cdots \quad \dfrac{d^N p(t)}{dt^N} \right]^T$

    then $\psi(t) = \boldsymbol{\rho}^T(\mathbf{x},t)\mathbf{b}$

    Define $\mathbf{A}(\mathbf{x},t) \equiv \begin{bmatrix} \mathbf{0} & \boldsymbol{\rho}^T(\mathbf{x},t) \\ \mathbf{0} & -\boldsymbol{\rho}^T(\mathbf{x},t) \end{bmatrix}$ and $\mathbf{f} \equiv -\delta_t [1 \quad 1]^T$

    Then $|\psi(t)| \leq \delta_t \;\; \forall t \in \Re \setminus T$ is equivalent to

    $\mathbf{A}(\mathbf{x},t)\mathbf{x} + \mathbf{f} \leq \mathbf{0} \;\; \forall t \in \Re \setminus T$

Proposed Method

Problem Formulation

Hence Problem $(\mathbf{P}'_f)$ becomes the following problem:

- Problem $(\mathbf{P}''_f)$

$$\min_{\mathbf{x}} \; J(\mathbf{x}) \equiv \frac{1}{2}\mathbf{x}^T\mathbf{Q}\mathbf{x} + \mathbf{c}^T\mathbf{x} + p$$

subject to

$$\mathbf{A}(\mathbf{x},t)\mathbf{x} + \mathbf{f} \le \mathbf{0} \quad \forall t \in \Re \setminus T$$

$$\frac{1}{2}\mathbf{x}^T\mathbf{Q}'(\omega)\mathbf{x} + \mathbf{c}'^T(\omega)\mathbf{x} + p'(\omega) \le 0 \quad \forall \omega \in \Re \setminus B$$

$$\mathrm{Re}\left(1 + \begin{bmatrix}\boldsymbol{\xi}^T(\omega) & \mathbf{0}\end{bmatrix}\mathbf{x}\right) < 0 \quad \forall \omega \in \Re$$

- Note that this is a nonlinear nonconvex continuous constrained optimization problem, which is difficult to solve.

Proposed Method

- Problem Formulation
  - Advantage: Problem $\left(\mathbf{P}_f''\right)$ solves both the numerator and the denominator coefficients of the frequency response of the pulse simultaneously.
  - The divergent problem due to the iterative design of the numerator and the denominator coefficients does not occur.
  - To solve this nonlinear and nonconvex problem, the bridging method is employed and a global optimal solution can be obtained if a solution exists.

Proposed Method

- Numerical Computer Simulation Result

$$B = [3.1GHz \quad 10.6GHz]$$

$$T = [0s \quad 1ns]$$

$$D(\omega) = \begin{cases} 10^{-9} & B \\ 0 & \Re \setminus B \end{cases}$$

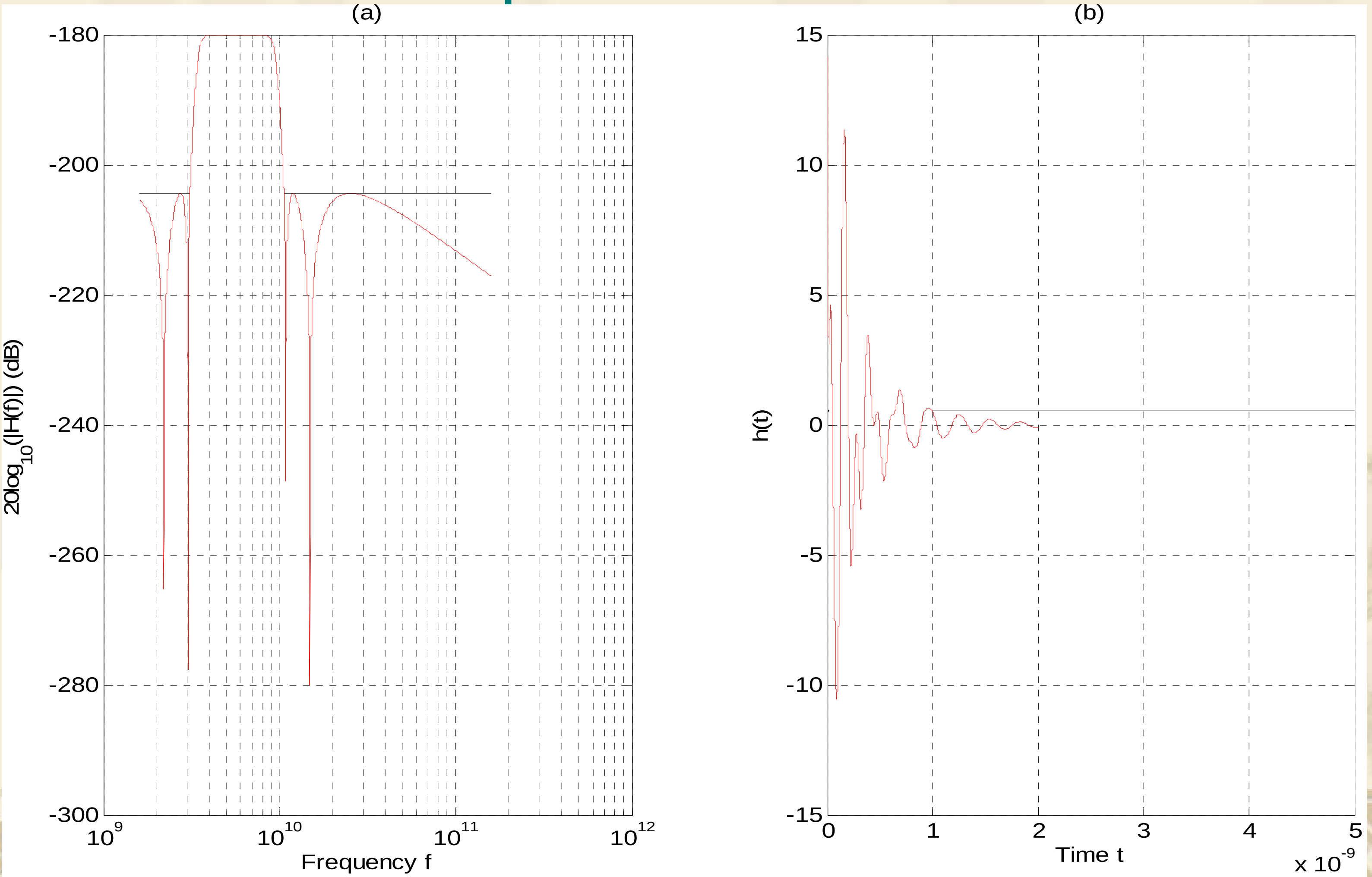
$$N = 10$$

$\delta_f = 6 \times 10^{-11}$ (6% of the peak values of the desired magnitude response)

$\delta_t = 0.5512$ (7.35% of the peak values of the desired impulse response)

Proposed Method

- Numerical Computer Simulation Result



Conclusions

- A pulse for UWB systems via an IIR filter design approach is proposed.
- Since a rational function is more general compared to standard pulses, better impulse and frequency responses of the pulse can be achieved.
- The numerator and the denominator coefficients are solved simultaneously, so the divergent problem due to the iterative design of the numerator and the denominator coefficients does not occur.

Conclusions

- The design of the numerator and the denominator coefficients of the frequency response of the pulse is formulated as a continuous constrained optimization problem subject to the requirements on both the time and frequency domains suggested by the FCC. Hence, the obtained solution is guaranteed to satisfy the required specifications.

Questions and Answers

Thank you!

Let me think…